**The University of Iowa**
**College of Liberal Arts and Sciences**
**Department of Statistics and Actuarial Science**
**ACTS:4380 (22S:183) Mathematics of Finance II**
**1:30 - 2:20PM MWF 40 SH Spring Semester 2015**

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Office Hours: Tuesday(1:30–2:20), Thursday(1:30–2:20), Friday(2:30–3:20), or by appointment
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Prerequisites: Grade of C+ or higher in ACTS:4130 (22S:174), and grade of C+ or higher in ACTS:3080 (22S:180) or ACTS:3085 (22S:179)

This course is intended for students who want to master the material in Exam MFE (Models for Financial Economics) of the Society of Actuaries (SOA). The **required textbook** is R.L. McDonald, *Derivatives Markets*, 3rd edition, Pearson. You may find it useful to bring the book to every class. The course will cover Chapters 9 to 14, 18, 20, 25, and parts of Chapters 19, 21, 23, and 24. Pearson has also published *Student Solutions Manual to Derivatives Markets* which provides solutions to even-numbered end-of-chapter questions in the textbook.
**Optional textbook**: J. Hull, *Options, Futures, and Other Derivatives*, Pearson. It is a textbook for a later SOA examination; earlier editions of it can be obtained cheaply.

There will be three evening examinations, to be held on February 20 (Friday), March 24 (Tuesday), and April 21 (Tuesday) in 40 SH. **You are to use an official CAS/SOA calculator during exams.** Tentatively, the final grade (H.W. 7% + Midterms 3×21% + Final 30%) will be assigned as follows:
Undergrads F[0, 40); D–[40, 45); D[45, 50); D+[50, 55); C–[55, 60); C[60, 65); C+[65, 70); B–[70, 75); B[75, 80); B+[80, 85); A–[85, 90); A[90, 95); A+[95, 100]
Graduate students F[0, 46); D–[46, 50.5); D[50.5, 55); D+[55, 59.5); C–[59.5, 64); C[64, 68.5); C+[68.5, 73); B–[73, 77.5); B[77.5, 82); B+[82, 86.5); A–[86.5, 91); A[91, 95.5); A+[95.5, 100]

In 2015, the dates for Exam MFE are March 13-19, July 9-15, and November 19-25.

ACTS:4380 (22S:183) Mathematics of Finance II
HW Set 1

Due at the beginning of class on Friday, January 23, 2015

**Write legibly. Show your steps. Explain your reasoning. Simplify your answers.**

1. (10 points) A stock pays dividends continuously. Between time t and time t+dt, dividends of amount 0.04S(t)dt will be paid, where S(t) denotes the price of one share of the stock at time t, $t \geq 0$. (Note that if the stock price is stochastic, the dividend payments are also stochastic.)
The current (t = 0) stock price is 100.
The continuously compounded risk-free interest rate, *r*, is 5%
What is the current price for the (stochastic) stream of dividends to be paid in the next 7 years?
**Hint**: Chapter 5 of McDonald (2013). Your answer does not depend on all information given.

2. The following is a variation of formula (14.16) on page 424 of McDonald (2013). (We shall derive this result later.)

$$V(s_1, s_2) = s_1 e^{-\delta_1 T} N\left( \frac{\ln\left[ \frac{s_1 e^{-\delta_1 T}}{s_2 e^{-\delta_2 T}} \right]}{\sigma\sqrt{T}} + \frac{1}{2}\sigma\sqrt{T} \right) - s_2 e^{-\delta_2 T} N\left( \frac{\ln\left[ \frac{s_1 e^{-\delta_1 T}}{s_2 e^{-\delta_2 T}} \right]}{\sigma\sqrt{T}} - \frac{1}{2}\sigma\sqrt{T} \right).$$

Here, $T$, $\delta_1$, $\delta_2$ and $\sigma$ are positive constants, and $N(x)$ is the cumulative standard normal distribution, $N(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{x} e^{-z^2/2} dz$.

(i) (10 points) **Calculate** the partial derivatives $\frac{\partial}{\partial s_1} V(s_1, s_2)$ and $\frac{\partial}{\partial s_2} V(s_1, s_2)$.

**Hint**: The variable $s_1$ appears in three places on the RHS of the formula for $V$. Hence, when you differentiate $V$ with respect to $s_1$, there will be three terms. However, the three terms can be simplified to one term because of a formula like the one on the very last line of page 379.
The partial derivative $\frac{\partial}{\partial s_2} V$ also only has one term after simplification.

(ii) (5 points) Evaluate $s_1 \frac{\partial}{\partial s_1} V(s_1, s_2) + s_2 \frac{\partial}{\partial s_2} V(s_1, s_2) - V(s_1, s_2)$.

**Hint**: If you know calculus well, you do not even need to use the results in (i). Just cite a theorem and show that the hypothesis of the theorem is satisfied.

(iii) (5 points) Evaluate the limit $\lim_{T \to 0} V(s_1, s_2)$.

**Hint**: You do not need to use the results in (i) and (ii). The answer is a function of $s_1$ and $s_2$, but does not depend on $\delta_1$, $\delta_2$ or $\sigma$.

**Remark**: With the *prepaid forward price* notation, the function $V$ can be expressed as

$$V(S_1(0), S_2(0)) = F^P_{0,T}(S_1) N\left( \frac{\ln\left[ \frac{F^P_{0,T}(S_1)}{F^P_{0,T}(S_2)} \right]}{\sigma\sqrt{T}} + \frac{1}{2}\sigma\sqrt{T} \right) - F^P_{0,T}(S_2) N\left( \frac{\ln\left[ \frac{F^P_{0,T}(S_1)}{F^P_{0,T}(S_2)} \right]}{\sigma\sqrt{T}} - \frac{1}{2}\sigma\sqrt{T} \right).$$

3. (10 points) Let $X$ be a random variable for which the moment generating function,

$$M_X(t) = \mathrm{E}[e^{tX}],$$

exists. Let $\mu$ and $\sigma^2$ denote the mean and variance of $X$, respectively.

For j = 3, 4, ... , let $\mu_j = \mathrm{E}[(X - \mu)^j]$ be the j-th central moment of $X$.

Consider the **Maclaurin series** (i.e., Taylor series expanded about the origin)

$$\ln(M_X(t)) = \kappa_0 + \kappa_1 \frac{t}{1!} + \kappa_2 \frac{t^2}{2!} + \kappa_3 \frac{t^3}{3!} + + \kappa_4 \frac{t^4}{4!} + ....$$

Determine the first five coefficients, $\kappa_0$, $\kappa_1$, … , $\kappa_4$, in terms of $\mu$, $\sigma^2$, $\mu_3$ and $\mu_4$.

**Remark**: Because ln means logarithm to the base *e*, the series expansion above can also be written as

$$M_X(t) = \exp\left( \kappa_0 + \kappa_1 \frac{t}{1!} + \kappa_2 \frac{t^2}{2!} + \kappa_3 \frac{t^3}{3!} + + \kappa_4 \frac{t^4}{4!} + .... \right).$$

**Administrative Home**

The College of Liberal Arts and Sciences is the administrative home of this course and governs matters such as the add/drop deadlines, the second-grade-only option, and other related issues. Different colleges may have different policies. Questions may be addressed to 120 Schaeffer Hall, or see the CLAS Academic Policies Handbook.

**Electronic Communication**

University policy specifies that students are responsible for all official correspondences sent to their University of Iowa e-mail address (@uiowa.edu). Faculty and students should use this account for correspondence (Operations Manual, III.15.2. Scroll down to k.11).

**Accommodations for Disabilities**

A student seeking academic accommodations should first register with Student Disability Services and then meet with the course instructor privately in the instructor's office to make particular arrangements. See http://sds.studentlife.uiowa.edu/ for more information.

**Academic Honesty**

All students taking CLAS courses have, in essence, agreed to the College's Code of Academic Honesty: "I pledge to do my own academic work and to excel to the best of my abilities, upholding the IOWA Challenge. I promise not to lie about my academic work, to cheat, or to steal the words or ideas of others; nor will I help fellow students to violate the Code of Academic Honesty." Any student committing academic misconduct is reported to the College and placed on disciplinary probation or may be suspended or expelled (CLAS Academic Policies Handbook).

**CLAS Final Examination Policies**

The date and time of every final examination is announced by the Registrar generally by the tenth day of classes. **No exams of any kind are allowed during the last week of classes.** All students should plan on being at the UI through the final examination period. It is the student's responsibility to know the date, time, and place of the final exam.

**Making a Suggestion or a Complaint**

Students with a suggestion or complaint should first visit with the instructor (and the course supervisor), and then with the departmental DEO. Complaints must be made within six months of the incident (CLAS Academic Policies Handbook).

**Understanding Sexual Harassment**

Sexual harassment subverts the mission of the University and threatens the well-being of students, faculty, and staff. All members of the UI community have a responsibility to uphold this mission and to contribute to a safe environment that enhances learning. Incidents of sexual harassment should be reported immediately. See the UI Comprehensive Guide on Sexual Harassment for assistance, definitions, and the full University policy.

**Reacting Safely to Severe Weather**

In severe weather, class members should seek appropriate shelter immediately, leaving the classroom if necessary. The class will continue if possible when the event is over. For more information on Hawk Alert and the siren warning system, visit the Public Safety website.